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The tensor Goldstone multiplet for partially broken supersymmetry

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Abstract

We show that the tensor gauge multiplet of $N = 1$ supersymmetry can serve as the Goldstone multiplet for partially broken rigid $N = 2$ supersymmetry. We exploit a remarkable analogy with the Goldstone-Maxwell multiplet of [1] to find its nonlinear transformation law and its invariant Goldstone action. We demonstrate that the tensor multiplet has two dualities. The first transforms it into the chiral Goldstone multiplet; the other leaves it invariant.

To the memory of Viktor I. Ogievetsky

1. As with any continuous symmetry, the spontaneous breaking of supersymmetry implies the existence of a Goldstone field. For the partial breaking of extended supersymmetry, this field – a spin-1/2 Goldstone fermion $\psi_\alpha(x)$ – belongs to a massless multiplet of the residual unbroken supersymmetry.

Curiously enough, the choice of Goldstone multiplet is not unique. For example, when $N=2$ supersymmetry is spontaneously broken to $N=1$, the Goldstone field can be a member of a chiral $N=1$ multiplet [2]. (This corresponds to the supermembrane of ref. [3]). In a recent paper [1], we demonstrated that the Goldstone field can also be contained in the $N=1$ Maxwell spin-(1/2,1) multiplet. We derived its nonlinear transformation under the spontaneously broken supersymmetry and constructed the invariant action. We found the Goldstone action to be duality invariant, and its spin-1 part to be precisely the Born-Infeld action [4].

In this letter we will show that there is yet another Goldstone multiplet for partially broken $N=2$ supersymmetry – the $N=1$ tensor gauge multiplet (whose field strength is the $N=1$ linear multiplet) [5]. The superpartners of the Goldstone fermion are a real scalar field $\ell(x)$ and a real antisymmetric tensor gauge field E_{mn}. We will see that the $N=1$ description of this multiplet is almost identical to that of the Maxwell multiplet. Using this fact and results of [1], we will derive the second, nonlinear supersymmetry, as well as the invariant action for the tensor Goldstone multiplet. We shall see that the tensor multiplet has two different dualities. One relates it to the chiral Goldstone multiplet of [2]; the other maps it to itself. We will conclude with a brief discussion of some unanswered questions, especially those relating to higher symmetries associated with the tensor Goldstone multiplet.

2. The $N=1$ tensor multiplet is usually described by a real scalar superfield $L(x,\theta,\bar\theta)$, constrained by

$$D^2 L = \bar D^2 L = 0. \tag{1}$$

Its independent components are the $\theta=0$ projections of L, $D_\alpha L$ and $[D_\alpha, \bar D_{\dot\alpha}]L$. They correspond to a scalar $\ell(x)$, a fermion $\psi_\alpha(x)$ and the field strength of a gauge tensor $V^m = \frac{1}{2}\epsilon^{mnkl}\partial_n E_{kl}$.

The solution to (1) is given by

$$L = D^\alpha \phi_\alpha + \bar D_{\dot\alpha}\bar\phi^{\dot\alpha}\ , \tag{2}$$

where ϕ_α is an arbitrary chiral $N=1$ superfield, $\bar D_{\dot\alpha}\phi_\alpha = 0$. For our purposes, we find it more instructive to describe the tensor multiplet in terms of a *spinor* superfield ψ_α,

$$\psi_\alpha = i D_\alpha L. \tag{3}$$

Using (1), we see that ψ_α is *antichiral* [6],

$$D_\beta \psi_\alpha = 0. \tag{4}$$

In addition it satisfies the constraint

$$\bar D^2 \psi_\alpha - 4i\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} = 0. \tag{5}$$

Equations (4) and (5) are equivalent to (1); they are irreducibility conditions for the superspin-1/2 representation of $N=1$ supersymmetry.

The free action of the tensor multiplet can be written as

$$S_{\rm free}(\psi) = \frac{1}{4}\int d^4x d^2\theta\, \bar{\psi}^2 + \frac{1}{4}\int d^4x d^2\bar{\theta}\, \psi^2. \tag{6}$$

In this spinorial formulation, the $N = 1$ tensor multiplet displays a remarkable similarity to the $N = 1$ Maxwell multiplet. As is well known, the latter is described by a *chiral* spinor superfield W_α,

$$\bar{D}_{\dot{\alpha}} W_\alpha = \; 0, \tag{7}$$

which satisfies the reality constraint

$$D^\alpha W_\alpha + \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} = \; 0. \tag{8}$$

This reality constraint (8) can be rewritten as an irreducibility condition,

$$D^2 W_\alpha - 4i\partial_{\alpha\dot{\alpha}} \bar{W}^{\dot{\alpha}} = \; 0. \tag{9}$$

The free action of the Maxwell multiplet reads

$$S_{\rm free}(W) = \frac{1}{4}\int d^4x d^2\theta\, W^2 + \frac{1}{4}\int d^4x d^2\bar{\theta}\, \bar{W}^2. \tag{10}$$

Comparing equations (4), (5), (6) with (7), (9), (10), we see a very close analogy between the tensor and Maxwell multiplets. One theory is related to the other by

(i) switching chirality in $N = 1$ superspace:

$$x^m \to x^m, \;\; \theta^\alpha \to \bar{\theta}^{\dot{\alpha}}, \;\; \bar{\theta}^{\dot{\alpha}} \to \theta^\alpha, \tag{11}$$

which maps the chiral $N = 1$ superspace $(x^m - 2i\theta\sigma^m\bar{\theta}, \theta^\alpha)$ to the antichiral superspace $(x^m + 2i\theta\sigma^m\bar{\theta}, \bar{\theta}^{\dot{\alpha}})$; and

(ii) keeping spinor superfield ψ_α inert:

$$\psi_\alpha(x - 2i\theta\sigma\bar{\theta}, \theta) \to \psi_\alpha(x + 2i\theta\sigma\bar{\theta}, \bar{\theta}). \tag{12}$$

In what follows we will use this analogy to establish the main features of the Goldstone tensor multiplet.

3. To find the broken supersymmetry and invariant action for the tensor multiplet, we extract the corresponding features of the Goldstone-Maxwell multiplet from [1] and then switch chirality.

Using this technique, we find that the second supersymmetry for the tensor multiplet is given by

$$\delta\psi_\alpha = \eta_\alpha - \frac{1}{4}D^2\bar{X}\eta_\alpha - i\partial_{\alpha\dot{\alpha}}X\bar{\eta}^{\dot{\alpha}}, \tag{13}$$

which implies

$$\delta L = i(\bar{\theta}\bar{\eta} - \theta\eta) - \frac{i}{2}D^\alpha\bar{X}\eta_\alpha + \frac{i}{2}\bar{D}_{\dot{\alpha}}X\bar{\eta}^{\dot{\alpha}}. \tag{14}$$

Here X is an antichiral $N = 1$ superfield, $D_\alpha X = 0$, which satisfies the recursive equation

$$X = \frac{\psi^2}{1 - \frac{1}{4}D^2\bar{X}}, \tag{15}$$

with the solution

$$
\begin{aligned}
X &= \psi^2 + \frac{1}{2}D^2\left[\frac{\psi^2\bar{\psi}^2}{1-\frac{1}{2}A + \sqrt{1-A+\frac{1}{4}B^2}}\right]. && (16)\\
A &= \frac{1}{2}(\bar{D}^2\psi^2 + D^2\bar{\psi}^2), \\
B &= \frac{1}{2}(\bar{D}^2\psi^2 - D^2\bar{\psi}^2). && (17)
\end{aligned}
$$

Under the second supersymmetry, X transforms as follows,

$$\delta X = 2\psi\eta. \tag{18}$$

The transformations (13) and (18) are consistent with the contraints imposed on the superfields; they close into the $N=2$ supersymmetry algebra.

Using these results, it is not hard to show that the invariant Goldstone action is given by

$$S_{\mathrm{tensor}} = \frac{1}{4}\int d^4x d^2\theta\, \bar{X} + \frac{1}{4}\int d^4x d^2\bar{\theta}\, X. \tag{19}$$

This is just the chirality-switched Goldstone-Maxwell action. Note that the bosonic part of the action is

$$S_{\mathrm{Bose}} = \int d^4x \left[1 - \sqrt{1 - (\partial^m\ell\partial_m\ell - V^mV_m) - (V^m\partial_m\ell)^2}\,\right]. \tag{20}$$

If V_m were zero, this would be the action of a 3-brane in five dimensions.

4. The tensor multiplet enjoys two different dualities. At the level of the bosonic fields, they can be understood as follows. In four dimensions, a tensor gauge field is dual to a scalar field and vice versa. Given a real tensor gauge field E_{mn}, one can dualize to a real scalar field $\tilde{\ell}$ by relaxing the constraint on its field strength V^m, $\partial_m V^m = 0$, and adding the Lagrange multiplier

$$\int d^4x\, V^m\partial_m\tilde{\ell}. \tag{21}$$

Alternatively, one can transform from a real scalar ℓ, with "field strength" $\ell_m = \partial_m\ell$, to a tensor gauge field $\tilde{E}_{mn}$ by relaxing the constraint $\partial_{[m}\ell_{n]} = 0$, and adding the Lagrange multiplier

$$\int d^4x\, \epsilon^{mnkl}\tilde{E}_{kl}\partial_{[m}\ell_{n]}. \tag{22}$$

Using these techniques, one can dualize the field strength V_m to a scalar field in the bosonic part of the Goldstone action (20). This leads to a dual action with two physical scalar fields. Alternatively, one can dualize the scalar field ℓ *and* the tensor field strength V_m. The dual action has the same field content – a scalar and a gauge tensor – as the original action. One can check that the first duality transforms (20) into the action for a 3-brane in six dimensions, while the second leaves it invariant.

In superspace, the second duality can be demonstrated using the spinorial representation for the tensor multiplet, and relaxing the irreducibility constraint (5) while keeping ψ_α antichiral. This is done by introducing the Lagrange multiplier

$$i\int d^4x d^2\theta\, \widetilde{\psi}^\alpha\psi_\alpha - i\int d^4x d^2\bar{\theta}\, \widetilde{\overline{\psi}}_{\dot{\alpha}}\overline{\psi}^{\dot{\alpha}}, \tag{23}$$

where $\widetilde{\psi}^\alpha$ is an antichiral superfield, subject to the irreducibility constraint (5),

$$\bar{D}^2\widetilde{\psi}_\alpha - 4i\partial_{\alpha\dot{\alpha}}\overline{\widetilde{\psi}}^{\dot{\alpha}} = 0. \tag{24}$$

The relaxed action (19), (23) has exactly the same form as the relaxed action for the Goldstone-Maxwell multiplet [1] (after switching chirality). Therefore, as in [1], varying with respect to ψ and substituting back produces the dual action

$$S(\widetilde{\psi}) = \frac{1}{4}\int d^4x d^2\theta\, \overline{\widetilde{X}} + \frac{1}{4}\int d^4x d^2\bar{\theta}\, \widetilde{X}, \tag{25}$$

where

$$\widetilde{X} = \frac{\widetilde{\psi}^2}{1-\frac{1}{4}D^2\overline{\widetilde{X}}}. \tag{26}$$

The dual action has exactly the same form as the original action (19).

The superspace form of the first duality maps the $N=1$ tensor multiplet into its chiral counterpart. The duality is implemented by rewriting the action (19) in terms of the real superfield L, and then relaxing the constraint (1) by adding the Lagrange multiplier

$$\int d^4x d^4\theta\, L(\phi+\bar{\phi}), \tag{27}$$

where ϕ is a chiral superfield, $\bar{D}_{\dot{\alpha}}\phi = 0$. If one varies the relaxed action with respect to L and substitutes back, one obtains the full nonlinear action for the chiral Goldstone multiplet [7],

$$S_{\rm dual}(\phi) \;=\; \int d^4x d^4\theta\; \mathcal{L}(\phi,\bar{\phi}), \tag{28}$$

where

$$\mathcal{L}(\phi,\bar{\phi}) \;=\; \phi\bar{\phi} + \frac{1}{8}\,D^\alpha\phi D_\alpha\phi \bar{D}_{\dot{\alpha}}\bar{\phi}\bar{D}^{\dot{\alpha}}\bar{\phi}\, f \tag{29}$$

and

$$\begin{aligned} f^{-1} &= 1+\frac{A}{2}+\sqrt{1+A+B}, \\ A &= -4\partial_m\phi\partial^m\bar{\phi} - \frac{1}{4}D^2\phi\bar{D}^2\bar{\phi}, \\ B &= 4(\partial_m\phi\partial^m\bar{\phi})^2 - 4(\partial_m\phi)^2(\partial^n\bar{\phi})^2. \end{aligned} \tag{30}$$

The action is invariant under the full nonlinear second supersymmetry,

$$\delta\phi \;=\; -2i\theta\eta - \frac{i}{4}\eta^\alpha\bar{D}^2D_\alpha\mathcal{L}. \tag{31}$$

This transformation closes into the off-shell $N=2$ supersymmetry algebra. Changing variables as follows,

$$\phi \;=\; \varphi + \frac{1}{16}\bar{D}^2(\bar{\varphi}D^\alpha\varphi D_\alpha\varphi) + O(\varphi^5) \tag{32}$$

one can show that the leading terms of the action (28) are precisely those of the chiral Goldstone action derived in [2].

It is interesting to note that the superfield $\mathcal{L}$ plays the role of an $N=1$ Maxwell prepotential. Indeed, if we define the field strength,

$$\mathcal{W}_\alpha \;=\; -\frac{i}{8}\bar{D}^2D_\alpha\mathcal{L}, \tag{33}$$

we find that ϕ and $\mathcal{W}$ obey the following $N = 2$ transformation laws,

$$
\begin{aligned}
\delta\phi &= -2i\theta\eta + 2\mathcal{W}\eta, \\
\delta\mathcal{W}_\alpha &= -\frac{1}{4}\bar{D}^2\bar{\phi}\,\eta_\alpha - i\partial_{\alpha\dot{\alpha}}\phi\,\bar{\eta}^{\dot{\alpha}}.
\end{aligned} \tag{34}
$$

We see that the Goldstone field ϕ and its $N = 1$ superfield Lagrangian are elements of an $N = 2$ vector multiplet. Together they form a *linear* representation of $N = 2$ supersymmetry – up to a field-independent shift.

In fact, the same holds true for the other Goldstone multiplets as well. Comparing (34) with eqs. (27) and (28) of ref. [1], and with eqs. (13) and (18) above, we see that the nonlinear transformation laws take a similar form. The vector multiplet Goldstone field, W_α, and its superfield Lagrangian, X, combine to form an vector $N = 2$ multiplet. Their transformations are identical to (34) (ignoring the shifts, and replacing ϕ by X and $\mathcal{W}_\alpha$ by W_α). The tensor multiplet Goldstone superfield, ψ_α, and its Lagrangian, X, are the elements of an $N = 2$ tensor multiplet.

5. The analogy between the $N = 1$ tensor gauge and Maxwell multiplets is closely related to an early work of Ogievetsky and Polubarinov [8], in which the authors introduced an antisymmetric tensor gauge field (the "notoph"), which is complementary to the photon. The notoph also has three degrees of freedom, but propagates with helicity zero on shell. Our chirality flipping procedure takes the $N = 1$ Maxwell multiplet into the $N = 1$ tensor gauge multiplet, which is the supersymmetric generalization of the notoph.

By construction, the Goldstone action (19) is invariant under $N = 2$ supersymmetry. It turns out that this action is also invariant under a full five-dimensional Poincaré supersymmetry. The scalar $\ell(x)$ is the Goldstone boson associated with the momentum in the fifth dimension; from a four-dimensional point of view, it is the Goldstone boson for a real central charge (as can be seen from (14)). Furthermore, the gradient $\partial_m\ell(x)$ can be shown to parametrize the Lorentz group coset $SO(1,4)/SO(1,3)$.

The fact that $\ell(x)$ and $\psi_\alpha(x)$ are both Goldstone fields leads one to speculate that the antisymmetric tensor gauge field might itself be a Goldstone field. If so, it would be interesting to understand its role from the algebraic and p-brane points of view.

This work completes a series of three papers in which we showed that the Goldstone fermion from the spontaneous breaking of $N = 2$ supersymmetry can be an element of an $N = 1$ chiral, vector or tensor multiplet. However, these papers do not answer the most intriguing question of all: Why are there three such multiplets, when only one would do?

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